$m \sim \text{Poisson}(\lambda)$, then $X_k = \#$ balls in bin k,

$$\Pr(X_1=x_1,\dots,X_6=x_6,\ m=x_1+\dots+x_6) = \Bigg(\overbrace{\frac{m!}{\prod_{i=1}^{6} x_i!}}^{\text{permutation}} \underbrace{\prod_{i=1}^{6} p_i^{x_i}}_{\text{indep.}} \Bigg) \times \Bigg(\overbrace{\frac{\lambda^m e^{-\lambda}}{m!}}^{\text{Poisson pdf}} \Bigg)$$

$$= \prod_{i=1}^{6} \frac{(\lambda p_i)^{x_i} e^{-\lambda p_i}}{x_i!} = \text{Prob}(\text{Pois}(\lambda p_1)=x_1,\dots,\text{Pois}(\lambda p_6)=x_6)$$

so by throwing Poisson # of rolls, then joint distribution of balls in each bin follows [independent poissons]

with $X_i \sim \text{Pois}(\lambda p_i)$, $p_i = \text{prob}(\text{ball in bin } i) = \frac{1}{6}$.

To use this to calculate, let $\mu = \lambda/6$, so that

$$\overbrace{Y_m}^{\#\text{ distinct, } m \text{ throws}} \triangleq \mathbb{1}_{\{X_1 \neq 0\}} + \mathbb{1}_{\{X_2 \neq 0\}} + \dots + \mathbb{1}_{\{X_6 \neq 0\}}$$

so again, using probability generating function,

$$E(t^{Y_m}) = E\left[t^{\mathbb{1}_{\{X_1 \neq 0\}}}\right]^6 = \left[e^{-\mu} + (1-e^{-\mu})t\right]^6$$

Moreover, we can decompose using law of total expectation

$$E[t^{Y_m}] = \sum_{k=0}^{\infty} E[t^{Y_m} \mid m=k]\, \text{Prob}(m=k)$$

$$= \sum_{k=0}^{\infty} E[t^{Y_m} \mid m=k] \left(\lambda^k e^{-\lambda}/k!\right)$$

In particular, $$\left\{ E[t^{Y_m}] e^{\lambda} \right\}_{t^4 \lambda^6 \text{ coef}} = \frac{\text{Prob}(Y_6=4)}{6!}$$

now, all that remains is to do that calculation of prob. generating function,

$$P(Y_6=4) = \left\{ 6!\, e^{-\lambda} \left[e^{-\frac{\lambda}{6}} + (1-e^{-\lambda/6})t \right]^6 \right\}_{\lambda^6 t^4 \text{ coef.}}$$

$$= 6! \left\{ \left[1 + (e^{\lambda/6}-1)t \right]^6 \right\}_{\lambda^6 t^4 \text{ coef.}}$$

$$= 6! \left\{ \binom{6}{4} (e^{\lambda/6}-1)^4 \right\}_{\lambda^6 \text{ coef.}}$$

$$= \frac{6!\,6!}{4!\,2!} \left\{ e^{\frac{4\lambda}{6}} - 4e^{\frac{3\lambda}{6}} + 6e^{2\lambda/6} - 4e^{\lambda/6} \right\}_{\lambda^6 \text{ coef.}}$$

$$= \frac{6!\,6!}{4!\,2!} \left[\frac{(4/6)^6}{6!} - 4\frac{(3/6)^6}{6!} + 6\frac{(2/6)^6}{6!} - 4\frac{(1/6)^6}{6!} \right] \approx \boxed{0.515}$$